Online Appendix for "Declining Business Dynamism and Worker Mobility"

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A Data Description

A.1 Quarterly Workforce Indicators (QWI)

The Longitudinal Employer Household Dynamics (LEHD) database is a linked employer-employee dataset constructed from state administrative records and maintained by the U.S. Census Bureau. While access to the underlying microdata in the LEHD is restricted, the Census Bureau publishes tabulations of the data at different levels of aggregation such as industry, geography, firm size and age, as well as worker demographics. In particular, the Census maintains the Quarterly Workforce Indicators (QWI), which contain information on hires, separations, turnover, employment growth, and earnings by industry, worker demographics, and firm age and size. The data can be downloaded from the webpage: https://lehd.ces.census.gov/data/#qwi.

A.2 Business Dynamics Statistics (BDS)

The Business Dynamics Statistics (BDS) datasets are maintained by the U.S. Census Bureau and contain annual measures of business dynamics such as job creation, job destruction, establishment births and deaths, and firm startups and exits. The data are available for the overall economy as well as by different establishment and firm characteristics. The BDS is derived from the Census Bureau's Longitudinal Business Database (LBD), a census of business establishments and firms in the U.S. with paid employees comprised of survey and administrative records. Data may be downloaded from https://www.census.gov/data/datasets/time-series/econ/bds/bds-datasets.html.

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A.3 Job-to-Job Flows (J2J)

To complement the QWI, the Census Bureau publishes additional detail on worker flows in the Job-to-Job Flows (J2J) database. The tabulations are similar to those in the QWI and statistics are available by firm characteristics (industry, age, and size) and by worker demographics (sex by age, sex by education, and race by ethnicity). These data contain measures of *direct* job-to-job transitions across employers and also allow to distinguish hires from other firms (poaching) from hires from the unemployment pool. They also allow to distinguish separations to another firm (job-to-job separations) from separations to nonemployment. The data can be downloaded from the webpage: https://lehd.ces.census.gov/data/#j2j.

A.4 Current Population Survey (CPS)

Annual Social and Economic Supplement (ASEC) To construct the measure of employer switching, I use data from the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS). The ASEC is based on a survey of more than 75,000 U.S. households and contains detailed questions on the social and economic characteristics of each person who is a household member as of the interview date. Questions in the survey pertain to the previous calendar year.

To construct the measure of employer switching used in the paper, I use a variable in ASEC that records the responses to the following survey question: "For how many employers did (name/you) work in [year]? If more than one at the same time, only count it as one employer." Since the question asks respondents to count simultaneous employment at multiple firms as only one employer, any respondent who answers that she had more than one employer in a given year must have switched jobs between firms at some point during that year. The employer switching rate is then estimated as the number of respondents who had more than one employer divided by total employment.[1]

This approach follows Molloy et al. (2016), which is the first paper to my knowledge to construct this specific measure of employer switching. I download the variable NUMEMPS, which contains responses to the survey question above, from the IPUMS CPS website (Flood et al., 2022). I select wage and salary workers in the private sector who reported that they were employed or had a job during the previous calendar year. IPUMS CPS data are available at https://cps.ipums.org/cps/.

[1]In practice, I weight each observation using the weighting variable ASECWT provided by IPUMS CPS.

Longitudinally Linked CPS In order to construct measures of job finding and separation by worker age group, I follow the procedure described in Shimer (2012) to link respondents in the CPS Basic Monthly Survey (BMS) across months. I gather data from IPUMS CPS and link survey respondents across consecutive months using the unique identifier `CPSIDV` (Flood et al., 2022).[2] This variable includes linking criteria that ensures individuals match on age, sex, and race characteristics. After linking individuals, information on their employment status in each month allows me to construct flow probabilities. Specifically, I use the variable `EMPSTAT` to determine whether a given individual was employed (E), unemployed (U), or not-in-the-labor-force (N) in a particular month. I then compute weighted sums of the number of individuals who transition across labor market states using longitudinal weights provided by IPUMS CPS.

The monthly job finding probability $P(UE)_t$ is defined as the fraction of unemployed individuals in month $t-1$ who are employed in month t. The monthly job separation probability $P(EU)_t$ is defined as the fraction of employed individuals in month $t-1$ who are unemployed in month t. Formulas are given below.

$$P(UE)_t = \frac{\#(\text{Unemployed in month } t-1 \text{ who are Employed in month } t)}{\#(\text{Unemployed in month } t-1)}$$

$$P(EU)_t = \frac{\#(\text{Employed in month } t-1 \text{ who are Unemployed in month } t)}{\#(\text{Employed in month } t-1)}$$

The job finding and job separation rates by age group are simply constructed by applying the above formulas for the relevant age sub-sample.

A.5 Labor Force Statistics (LFS)

To construct the employment-to-population ratio and the labor force share of workers in each age group, I use data from the U.S. Bureau of Labor Statistics (BLS) Labor Force Statistics (LFS) database. The LFS contains statistics on U.S. labor force characteristics tabulated by different demographic groups such as age, race, sex, education, and marital status. I obtain the series listed in Table A.1 from the BLS website. The fraction of age 55 or older workers is simply the number of age 55 or older workers in the civilian labor force divided by the number of age 25 or older workers in the civilian labor force. The data are available at https://www.bls.gov/cps/.

[2] See the following link for more information on linking individuals across surveys in the IPUMS CPS data: https://cps.ipums.org/cps/cps_linking_documentation.shtml.

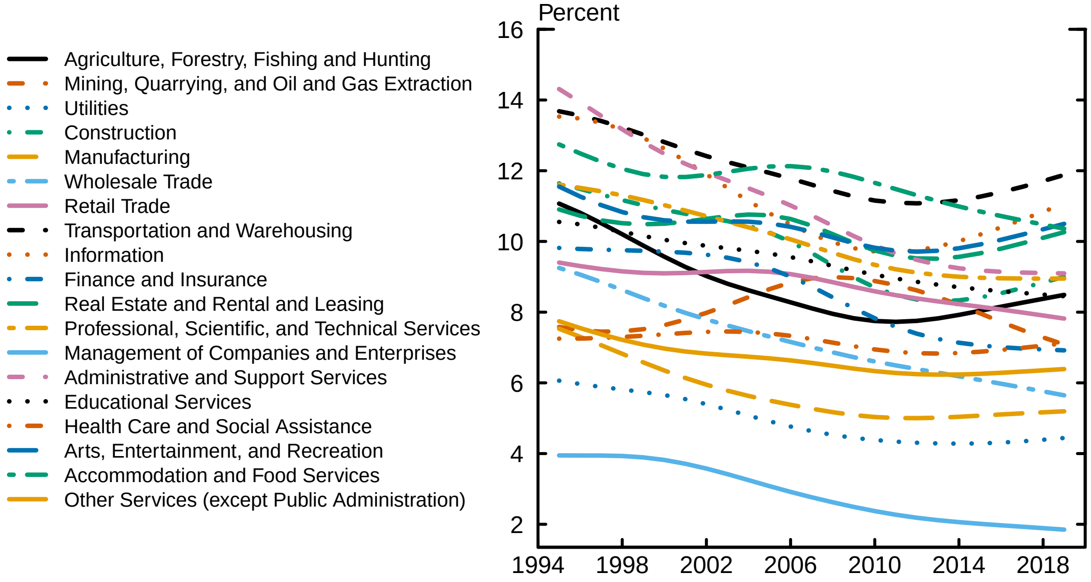
Table A.1: Variables in the LFS

Series ID	Labor Force Status	Demographic Group
LNS11000164	Civilian labor force	Men, age 25 to 34 years
LNS12300164	Employment-population ratio	Men, age 25 to 34 years
LNS11000173	Civilian labor force	Men, age 35 to 44 years
LNS12300173	Employment-population ratio	Men, age 35 to 44 years
LNS11000182	Civilian labor force	Men, age 45 to 54 years
LNS12300182	Employment-population ratio	Men, age 45 to 54 years
LNS11024231	Civilian labor force	Men, age 55 years and older
LNS12324231	Employment-population ratio	Men, age 55 years and older

Notes: Series are at the monthly frequency and are seasonally adjusted by the BLS.

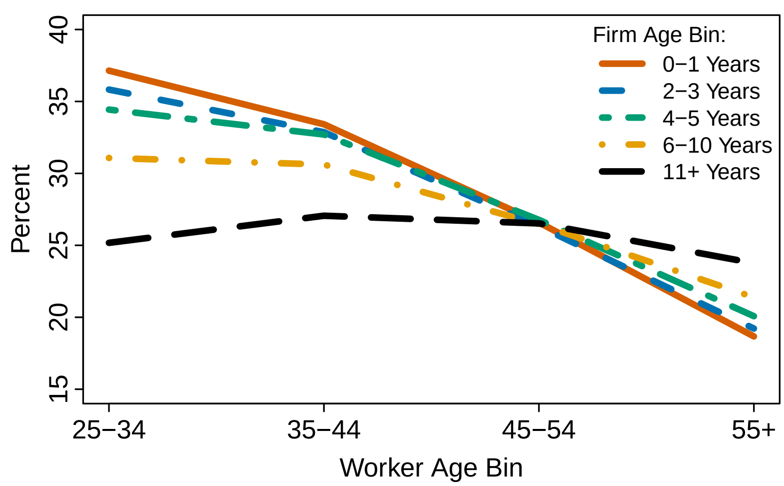
B Additional Figures

Figure B.1: Trends in Firm Entry Rate by Sector



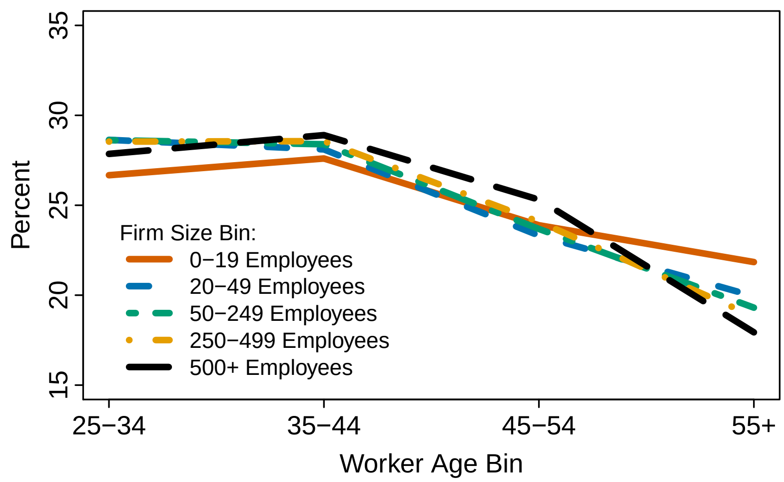
Notes: Entry rate defined as the number of age 0 firms divided by the total number of firms. Data are from the BDS. Series are HP-filtered with an annual smoothing parameter.

Figure B.2: Employment Distribution Across Worker Age by Firm Age, Granular Data



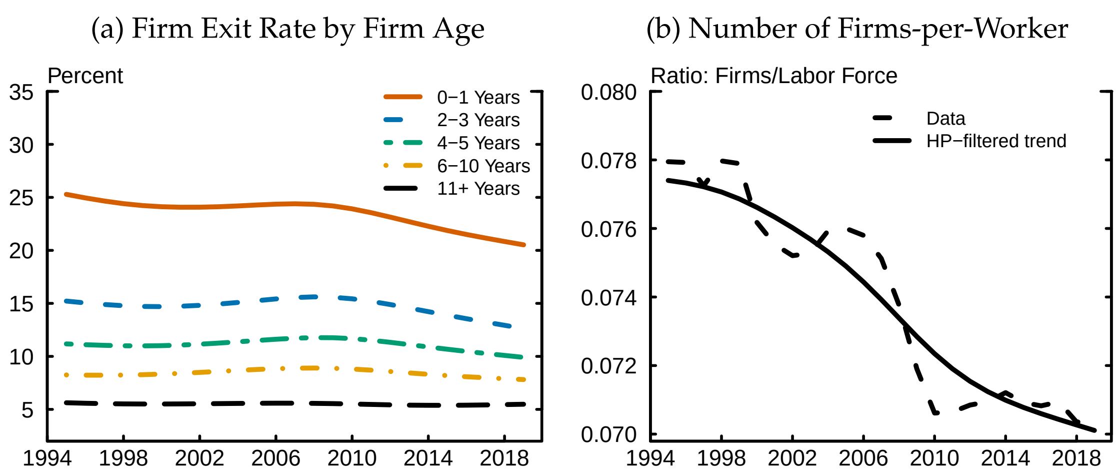
Notes: Figure shows average employment composition, in percentages, across worker age group for firms in different age groups. Data on employment by worker and firm age group are from the QWI. For all series, I include only male workers and take averages over state × industry × year cells.

Figure B.3: Employment Distribution Across Worker Age by Firm Size



Notes: Figure shows average employment composition, in percentages, across worker age group for firms in different size groups. Data on employment by worker age and firm size group are from the QWI. For all series, I include only male workers and take averages over 1994–2019.

Figure B.4: Calibrating the Law of Motion for the Mass of Firms



Notes: Firm exit rate for each firm age bin is defined as the number of firm deaths in the respective age bin divided by the total number of firms in the respective age bin. Data are from the BDS. The ratio of firms/labor force is defined in the same way as the model: the total number of firms in the economy divided by the total number of male workers over the age of 25 in the labor force. Data on the total number of firms in the economy is from the BDS. Data on the labor force is from the LFS. Series are HP-filtered with an annual smoothing parameter.

C Additional Empirical Results

In this section, I conduct robustness checks of the patterns documented in Section 2 in the main text. Table 1 shows that young firms (age 0-10 years) have an employment share of young workers (age 25-44 years) about 15 percentage points higher relative to mature firms (age 11 years or older). However, these sorting patterns may be driven by worker and firm characteristics other than age.

For instance, sorting between young workers and young firms may be driven by skill and productivity differences across these groups. A large literature on labor market sorting shows that workers with higher skill levels tend to match with firms with higher productivity levels (Lise and Robin, 2017). To the extent that older workers have been able to achieve higher education levels, and education proxies for worker skill, we may expect to see a higher share of older workers at older firms. Table C.1 explores this possibility.

Table C.1: Worker and Firm Sorting Patterns by Firm Age, Worker Education Groups

	(1)	(2)	(3)	(4)
Firm Age 0–10 Years	3.465***	1.364***	0.528***	−0.407***
	(0.025)	(0.024)	(0.049)	(0.078)
Frac. Age 25–44		0.065***	0.073***	0.073***
		(0.001)	(0.001)	(0.001)
ln(Avg. Firm Size)			−0.731***	−0.606***
			(0.032)	(0.033)
Firm Age 0–10 Years × ln(Avg. Firm Size)				0.466***
				(0.030)
Year Fixed Effects	X	X	X	X
State Fixed Effects	X	X	X	X
Industry Fixed Effects	X	X	X	X
Observations	430,334	420,191	367,033	367,033
R^2	0.739	0.815	0.814	0.815
Adjusted R^2	0.739	0.815	0.814	0.814

Notes: Sample includes only male workers age 25 and over for the years 1994–2019. Frac. Age 25-44 is the fraction of a firm's workforce between the ages of 25 and 44. ln(Avg. Firm Size) is the natural logarithm of average firm size. Industry fixed effects are at the 4-digit NAICS level. Standard errors in parentheses. $^{*}p \leq 0.10$;** $p \leq 0.05$;*** $p \leq 0.01$.

The table shows the results of regressions similar to those in main text, where instead the outcome variable is the fraction of a firm's work force with a high school education or less. From the first three columns of the table, we can see that younger firms employ, if anything, a slightly higher fraction of lower skilled workers, though the magnitude of this association reduces significantly after controlling for the fraction of young workers at

Table C.2: Worker and Firm Sorting Patterns by Firm Size, Worker Age Groups

	(1)	(2)	(3)	(4)
Firm Size < 500 Employees	2.448***	2.584***	2.307***	0.065**
	(0.041)	(0.040)	(0.035)	(0.032)
Frac. Educ. ≤ High School				0.179***
				(0.002)
Year Fixed Effects	X	X	X	X
State Fixed Effects		X	X	X
Industry Fixed Effects			X	X
Observations	425,682	425,682	425,682	388,622
R^2	0.143	0.175	0.400	0.486
Adjusted R^2	0.143	0.175	0.399	0.485

Notes: Sample includes only male workers age 25 and over for the years 1994–2019. Frac. Educ. ≤ High School is the fraction of a firm's workforce with less than or equal to a high school education. Industry fixed effects are at the 4-digit NAICS level. Standard errors in parentheses. $^{*}p \leq 0.10$;$^{**}\ p \leq 0.05$;$^{***}\ p \leq 0.01$.

a firm. Moreover, controlling for differences in firm size reveals that larger firms have a lower fraction of low skilled workers (Column (3)), but that this pattern is weaker among younger firms (Column (4)). Overall, the results are consistent with some degree of positive assortative matching between high skill workers and high productivity firms, but the firm life-cycle also plays a role; some small, yet highly productive young firms likely employ high skill workers in larger proportions. Lastly, the magnitudes of these sorting patterns are much smaller than those documented in Table 1.

To further explore the firm size dimension of worker and firm sorting patterns, I use an indicator for firm size instead of firm age as the independent variable of interest. Table C.2 displays the results. The table shows that smaller firms, on average, have a higher fraction of younger workers relative to firms with 500 employees or more. However, this pattern is largely driven by skill differences across worker age groups. Column (4) of the table shows that this association almost entirely disappears after controlling for differences in the employment share of low skill workers. Therefore, sorting on worker age and firm size likely results from the moderate degree of sorting on worker skill and firm size, as shown in Table C.1.

Lastly, Table C.3 explores worker skill and firm size sorting patterns directly. The outcome variable in this table is the fraction of workers with less than or equal to a high school education and the independent variable is an indicator for firm size instead of firm age. Here, we can see mostly clearly that even after controlling for differences in age composition across firm size categories, small firms employ a moderately higher fraction

Table C.3: Worker and Firm Sorting Patterns by Firm Size, Worker Education Groups

	(1)	(2)	(3)	(4)
Firm Size < 500 Employees	5.555***	5.668***	4.586***	3.507***
	(0.045)	(0.043)	(0.026)	(0.021)
Frac. Age 25–44				0.085***
				(0.001)
Year Fixed Effects	X	X	X	X
State Fixed Effects		X	X	X
Industry Fixed Effects			X	X
Observations	399,076	399,076	399,076	388,622
R^2	0.039	0.095	0.687	0.770

Notes: Sample includes only male workers age 25 and over for the years 1994–2019. Frac. Age 25-44 is the fraction of a firm's workforce between the ages of 25 and 44. Industry fixed effects are at the 4-digit NAICS level. Standard errors in parentheses. $^{*}p \leq 0.10;^{**} p \leq 0.05;^{***} p \leq 0.01$.

of lower skill workers. Again, these patterns are much less stable across different controls and of a much smaller magnitude than those displayed in Table 1. Therefore, I conclude that the sorting patterns between young firms and young workers are not simply masking differences in worker skill and firm productivity. Instead, the life-cycle component of employment sorting is accounted for by other forces, such as the joint dynamics of young workers and firms, or differences in where firms of different ages sit on the job ladder.

D Derivations and Proofs

To keep the notation simple, I normalize worker search intensity to 1 and abstract from retirement in the derivations below such that $\phi_x^i = \kappa_i \psi_x = 1$ and $\omega_x = 0 \; \forall x$. This is without loss of generality, and the same derivations hold in the case with differences in search intensity as well as retirement rates. I also suppress the terms in the expectations operator $\mathrm{E}[\cdot]$ to conserve on notation. For unemployed workers, expectations are over values of x' and for any joint value objects, expectations are over combinations of (x', y').

D.1 Unemployed Worker Value Function

The assumption that workers hired out of unemployment have zero bargaining power reduces the unemployed worker's value function to: $W_t^u(x) = b(x) + \beta \, \mathrm{E}\left[W_{t+1}^u(x')\right]$.

Proof. Start with the equation for the worker's value of unemployment.

$$W_t^u(x) = b(x) + \beta \, \mathrm{E}\left[(1-\lambda_{t+1})W_{t+1}^u(x') + \lambda_{t+1} \int \max\{W_{t+1}^e(x', y'), W_{t+1}^u(x')\} \frac{v_{t+1}(y')}{V_{t+1}} dy'\right]$$

Workers hired out of unemployment have zero bargaining power and therefore receive zero surplus share. In other words, firms are able to extract the entire match surplus upon matching with an unemployed worker. Therefore, workers hired out of unemployment simply receive the value of unemployment as their continuation value when matching with a firm. This implies that $W_t^e(x, y) \equiv W_t^e(x, y, 0) = W_t^u(x)$.[3] Substituting this into the equation above and reducing the expression yields the desired result.

$$\begin{aligned}
W_t^u(x) &= b(x) + \beta \, \mathrm{E}\left[(1-\lambda_{t+1})W_{t+1}^u(x') + \lambda_{t+1} \int \max\{W_{t+1}^e(x', y', 0), W_{t+1}^u(x')\} \frac{v_{t+1}(y')}{V_{t+1}} dy'\right] \\
&= b(x) + \beta \, \mathrm{E}\left[(1-\lambda_{t+1})W_{t+1}^u(x') + \lambda_{t+1} \int \max\{W_{t+1}^u(x'), W_{t+1}^u(x')\} \frac{v_{t+1}(y')}{V_{t+1}} dy'\right] \\
&= b(x) + \beta \, \mathrm{E}\left[(1-\lambda_{t+1})W_{t+1}^u(x') + \lambda_{t+1} \int W_{t+1}^u(x') \frac{v_{t+1}(y')}{V_{t+1}} dy'\right] \\
&= b(x) + \beta \, \mathrm{E}\left[(1-\lambda_{t+1})W_{t+1}^u(x') + \lambda_{t+1} W_{t+1}^u(x')\right] \\
&= b(x) + \beta \, \mathrm{E}\left[W_{t+1}^u(x')\right]
\end{aligned}$$

□

[3]We can also see this by setting $\sigma_t = 0$ in the definition of the employed worker's value function written in terms of the surplus share: $W_t^e(x, y, \sigma_t) = W_t^u(x) + \sigma_t S_t(x, y)$. See below for more details.

D.2 Joint Surplus Function

The joint surplus function is defined as the joint match value net of the unemployed worker's value, $S_t(x,y) \equiv P_t(x,y) - W_t^u(x)$. As mentioned in the text, the model is block recursive such that neither the distribution of firms in the economy nor the distribution of workers across matches enters the value function for the joint surplus.

Proof. First, start with the equation for the joint match value $P_t(x,y)$.

$$\begin{aligned}
P_t(x,y) = p(x,y) \\
&+ \beta\,\mathrm{E}\,\Big[\big(1-(1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\big)W_{t+1}^u(x') \\
&\quad + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\Big((1-\lambda_{t+1})P_{t+1}(x',y') \\
&\quad + \lambda_{t+1}\int \max\{P_{t+1}(x',y'), W_{t+1}^e(x',y'',y')\}\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y''\Big)\Big]
\end{aligned}$$

Due to the sequential auctions framework, the continuation value in the case that an employed worker contacts another firm is independent of the worker value $W_t^e(x,y,y')$. This is because there are two cases: either the worker moves to the poaching firm y' and extracts the entire match value (net of the outside option), or the worker stays at the incumbent firm and renegotiates their surplus share upwards in accordance with the value offered by the unsuccessful poaching firm. Therefore, $P_t(x,y) \geq W_t^e(x,y,y')$. We can use this expression to reduce the match value to the equation below.

$$\begin{aligned}
P_t(x,y) = p(x,y) \\
&+ \beta\,\mathrm{E}\Big[\big(1-(1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\big)W_{t+1}^u(x') \\
&\quad + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}P_{t+1}(x',y')\Big]
\end{aligned}$$

We then use the definition of the unemployed worker value function. As shown above, $W_t^u(x) = b(x) + \beta\,\mathrm{E}\left[W_{t+1}^u(x')\right]$. Therefore, subtracting $W_t^u(x)$ from both sides yields:

$$\begin{aligned}
P_t(x,y) - W_t^u(x) = p(x,y) - b(x) - \beta\,\mathrm{E}\left[W_{t+1}^u(x')\right] \\
&+ \beta\,\mathrm{E}\Big[\big(1-(1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\big)W_{t+1}^u(x') \\
&\quad + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}P_{t+1}(x',y')\Big].
\end{aligned}$$

Finally, rearranging and using the definition of the joint surplus yields the desired result.

$$
\begin{aligned}
P_t(x,y) - W_t^u(x) &= p(x,y) - b(x) \\
&\quad + \beta\,\mathrm{E}\Big[\Big(1-(1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\Big)W_{t+1}^u(x') \\
&\qquad + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}P_{t+1}(x',y') - W_{t+1}^u(x')\Big] \\
&= p(x,y) - b(x) \\
&\quad + (1-\delta_{x,y})\beta\,\mathrm{E}\Big[\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\Big(P_{t+1}(x',y') - W_{t+1}^u(x')\Big)\Big] \\
\implies S_t(x,y) &= p(x,y) - b(x) + (1-\delta_{x,y})\beta\,\mathrm{E}\Big[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\Big)\Big(S_{t+1}(x',y')\Big)\Big] \\
&= p(x,y) - b(x) + (1-\delta_{x,y})\beta\,\mathrm{E}\Big[\max\{S_{t+1}(x',y'),0\}\Big]
\end{aligned}
$$

□

D.3 Deriving the Wage Equation

We can use the definition of the surplus share in Equation 9 to represent the worker's value of employment as a function of the surplus and the surplus share.

$$W_t^e(x,y,\sigma_t) \equiv W_t^u(x) + \sigma_t S_t(x,y)$$

From this equation, we can explicitly see that hiring from unemployment entails setting $\sigma_t = 0$. Then, if a worker employed at some firm y meets another firm y', the surplus share σ_t evolves according to the piecewise function below.

$$
\sigma_t' = \begin{cases}
\frac{S_t(x,y)}{S_t(x,y')} & S_t(x,y') > S_t(x,y) \\
\frac{S_t(x,y')}{S_t(x,y)} & \sigma_t S_t(x,y) < S_t(x,y') \leq S_t(x,y) \\
\sigma_t & S_t(x,y') \leq \sigma_t S_t(x,y)
\end{cases}
$$

Notice that this expression mirrors the function $R(\cdot)$ in the main text. In the first case, the worker is poached and moves to firm y', extracting the entire surplus $S_t(x,y)$ of her previous match at firm y. In the second case, the worker stays at firm y, but renegotiates her surplus share to the full amount of the surplus $S_t(x,y')$ at firm y'. In the third case, the offer is below her current surplus share and is therefore too low to trigger a renegotiation;

the worker simply discards the offer and stays at firm y with the same surplus share.

Now, using the definition of the employed worker value $W_t^e(x,y,\sigma_t) = W_t^u(x) + \sigma_t S_t(x,y)$, we solve for a wage $w_t(x,y,\sigma_t)$ that implements this contract.

$$\begin{aligned}
W_t^e(x,y,\sigma_t) &= W_t^u(x) + \sigma_t S_t(x,y) \\
&= w_t(x,y,\sigma_t) + \beta \,\mathrm{E}\left[W_{t+1}^u(x')\right] \\
&\quad - (1-\delta_{x,y})\beta \,\mathrm{E}\Big[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\Big(\lambda_{t+1}\int Q_{t+1}(x',y',\sigma_{t+1},y'')\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y'' \\
&\qquad + (1-\lambda_{t+1})\sigma_{t+1}S_{t+1}(x',y')\Big)\Big]
\end{aligned}$$

where $Q_t(x,y,\sigma_t,y')$ is defined similarly to σ_t' above and represents the surplus the worker captures due to a renegotiation. In other words, it is the second best of the three values $\sigma_t S_t(x,y)$, $S_t(x,y')$, and $S_t(x,y)$.

$$Q_t(x,y,\sigma_t,y') = \begin{cases} S_t(x,y) & S_t(x,y') > S_t(x,y) \\ S_t(x,y') & \sigma_t S_t(x,y) < S_t(x,y') \leq S_t(x,y) \\ \sigma_t S_t(x,y) & S_t(x,y') \leq \sigma_t S_t(x,y) \end{cases}$$

Next, notice that from expression for the unemployed worker's value function, we have that $\beta\,\mathrm{E}\left[W_{t+1}^u(x')\right] = W_t^u(x) - b(x)$, so we can use this to eliminate $\beta\,\mathrm{E}\left[W_{t+1}^u(x')\right]$ and $W_t^u(x)$ from the above equation. We then have

$$\begin{aligned}
\sigma_t S_t(x,y) &= w_t(x,y,\sigma_t) - b(x) \\
&\quad - (1-\delta_{x,y})\beta \,\mathrm{E}\Big[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\Big(\lambda_{t+1}\int Q(x',y',\sigma_{t+1},y'')\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y'' \\
&\qquad + (1-\lambda_{t+1})\sigma_{t+1}S_{t+1}(x',y')\Big)\Big]
\end{aligned}$$

Lastly, we substitute the definition of the surplus equation into this equation and solve for $w_t(x,y,\sigma_t)$, which yields the desired result.

$$\begin{aligned}
w_t(x,y,\sigma_t) &= \sigma_t p(x,y) + (1-\sigma_t)b(x) \\
&\quad - (1-\delta_{x,y})\beta \,\mathrm{E}\left[\mathbb{1}\{S_{t+1}(x',y') \geq 0\} \cdot \lambda_{t+1}\int R_{t+1}(x',y',\sigma_{t+1},y'')\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y''\right]
\end{aligned}$$

where $R_t(x,y,\sigma_t,y') \equiv Q_t(x,y,\sigma_t,y') - \sigma_t S_t(x,y)$ is defined as below and represents the additional surplus the worker captures due to a renegotiation.

It is given by the piecewise function:

$$R_t(x,y,\sigma_t,y') = \begin{cases} S_t(x,y) - \sigma_t S_t(x,y) & S_t(x,y') > S_t(x,y) \\ S_t(x,y') - \sigma_t S_t(x,y) & \sigma_t S_t(x,y) < S_t(x,y') \leq S_t(x,y) \\ 0 & S_t(x,y') \leq \sigma_t S_t(x,y) \end{cases}$$

The first case corresponds to a situation where the worker is poached. In this case, she is able to capture the entire surplus from her old firm and therefore receives $S_t(x,y)$ net of the previous surplus share $\sigma_t S_t(x,y)$ in her old match. In the second case, the offer is higher than her previous outside offer, but not high enough to trigger a poaching event. The worker is able to renegotiate her surplus share at the incumbent firm in order to extract the full value of the outside offer. She therefore receives $S_t(x,y')$ net of her previous surplus share $\sigma_t S_t(x,y)$. In the third case, the outside offer is not sufficiently high to trigger a renegotiation and the offer is discarded.

D.4 Contract Distribution

Average wages by (x,y) pair are given by

$$w_t(x,y) = \int w_t(x,y,\sigma_t) g_t(x,y,\sigma_t) \,\mathrm{d}\,\sigma_t$$

where $w_t(x,y,\sigma_t)$ is the wage for a worker of age x employed at firm y with surplus share σ_t and $g_t(x,y,\sigma_t)$ is the distribution of σ's within (x,y) matches. Let $G_t(x,y,\sigma_t)$ be the cumulative distribution function corresponding to $g_t(x,y,\sigma_t)$. The contract distribution is defined similarly to the worker flow equations by the law of motion:

$$\begin{aligned} G_t(x,y,\sigma_t) = \tilde{G}_t(x,y,\sigma_t) &+ \lambda_t \int \tilde{e}_t(x,y') \frac{v_t(y)}{V_t} \mathbb{1}\{\sigma_t S_t(x,y) > S_t(x,y')\} \,\mathrm{d}\,y' \\ &- \lambda_t \int \tilde{G}_t(x,y,\sigma_t) \frac{v_t(y')}{V_t} \mathbb{1}\{\sigma_t S_t(x,y) < S_t(x,y')\} \,\mathrm{d}\,y' \\ &+ \lambda_t \,\tilde{u}_t(x) \frac{v_t(y)}{V_t} \mathbb{1}\{S_t(x,y) \geq 0\} \end{aligned}$$

where $\tilde{G}_t(x',y',\sigma_t) = \Pi_{x'|x} \cdot \Pi_{y'|y} \cdot (1-\delta_{x,y}) \cdot \mathbb{1}\{S_t(x,y) \geq 0\} \cdot G_{t-1}(x,y,\sigma_t)$.

E Additional Model Details

The transition matrices for worker age bin and firm age bin are given by the following expressions. Note that the model is set to monthly frequency.

$$\Pi_{x'|x} = \begin{bmatrix} 1-\frac{1}{120} & \frac{1}{120} & 0 & 0 \\ 0 & 1-\frac{1}{120} & \frac{1}{120} & 0 \\ 0 & 0 & 1-\frac{1}{120} & \frac{1}{120} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Pi_{y'|y} = \begin{bmatrix} 1-\frac{1}{24} & \frac{1}{24} & 0 & 0 & 0 \\ 0 & 1-\frac{1}{24} & \frac{1}{24} & 0 & 0 \\ 0 & 0 & 1-\frac{1}{24} & \frac{1}{24} & 0 \\ 0 & 0 & 0 & 1-\frac{1}{60} & \frac{1}{60} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

E.1 Model Solution

To compute the model solution, I use standard numerical techniques to solve the value function for the joint match surplus and to find the distribution of employment across worker and firm ages in steady state. Given values for $p(x, y)$, $b(x)$, and $\delta_{x,y}$, I first solve for the joint surplus function (Equation 2) by value function iteration. Then, I iterate on the worker flow equations (Equations 7 and 8) in order to solve for the steady state worker distribution, starting from an initial guess where all workers are unemployed. Each step of the iteration requires solving for aggregate search intensity (Equation 3), the value of a filled vacancy (Equation 4), and aggregate vacancies (Equation 6) in order to pin down the contact rates λ_t and μ_t. This also determines the vacancy distribution across firm ages $\frac{v_t(y)}{V_t}$. Next, I solve for wages at the match level by first using Equation 10 to obtain the wage $w_t(x, y, \sigma_t)$ for any pair (x, y) and any possible surplus share $\sigma_t = \sigma_t(x, y, y')$; then, I iterate on the law of motion for the distribution of contracts across σ_t within an (x, y) pair. This allows me to compute average wages by (x, y) pair. Appendix D.4 shows the law of motion for the distribution of wage contracts. With few worker and firm age bins, the entire solution algorithm converges very quickly.

F Additional Calibration Details

F.1 Constructing Data Moments

Table F.1 summarizes the data moments and their sources. Below, I provide additional detail about how I construct each moment in the data.

Table F.1: Data Moments

Moment	Bins	Source
Labor force share	Male workers age {25–34, 35–44, 45–54, 55+}	LFS
Job finding rate	Male workers age {25–34, 35–44, 45–54, 55+}	CPS
Job destruction rate	Firms age {0–1, 2–3, 4–5, 6–10, 11+}	BDS
Firms-per-worker	Firms age {0–1, 2–3, 4–5, 6–10, 11+}	BDS, LFS
Average firm size	Firms age {0–1, 2–3, 4–5, 6–10, 11+}	BDS
Average earnings	Male workers age {25–34, 35–44, 45–54, 55+} & Firms age {0–1, 2–3, 4–5, 6–10, 11+}	QWI
Employment share	Firms age {0–1, 2–3, 4–5, 6–10, 11+}	BDS
Poaching share	Firms age {0–1, 2–3, 4–5, 6–10, 11+}	J2J
Share workers < 45	Firms age {0–1, 2–3, 4–5, 6–10, 11+}	QWI
EU separation rate	Male workers age {25–34, 35–44, 45–54, 55+}	CPS
Job-to-job flow rate	Male workers age {25–34, 35–44, 45–54, 55+}	J2J

Notes: CPS data are from IPUMS CPS. LFS data are from the BLS website. I use the 2021 release of the BDS. I use the R2023Q4 releases of the QWI and J2J.

Labor force share To construct the labor force share by worker age group, I download the series in Table A.1 from the LFS at a seasonally adjusted, monthly frequency. The sample includes male workers in the age groups 25–34, 35–44, 45–54, and 55 and older. The size of the aggregate labor force is the sum across these groups. The labor force shares equal the size of the labor force in each group relative to the total.

Job finding rate I construct job finding rates in the CPS as described above. The sample includes only male workers age 25 and older. I take averages within age bins over 1990–1994 in order to set the search intensity by worker age bin parameters ψ_x. I target the average aggregate job finding rate over 1990–1994 in the moment matching exercise.

Job destruction rate I follow the BDS methodology and define the job destruction rate (JDR) as follows:[4]

$$JDR_{i,t} = \frac{\sum_{i \in s, g_{i,t}<0} (E_{i,t} - E_{i,t-1})}{0.5 * (E_{i,t} + E_{i,t-1})}$$

for establishments i in group s and where $g_{i,t} = (E_{i,t} - E_{i,t-1})/(0.5 * (E_{i,t} + E_{i,t-1}))$. Job destruction is the sum of all employment losses from contracting establishments from year t–1 to year t including establishments shutting down. As is common in the literature, the denominator normalizes across adjacent years. I download data by firm age bin ($s = \{$0–1, 2–3, 4–5, 6–10, 11+$\}$) from the BDS website and take averages over 1990–1994.

Firms-per-worker The number of firms by firm age bin is from the BDS. The size of the aggregate labor force is constructed as above from the LFS using only male workers age 25 and over. I take the ratio of the number of firms to the size of the aggregate labor force by firm age bin. I then HP-filter each series using an annual smoothing parameter ($\lambda = 100$). The steady state mass of firms by firm age bin $\bar{m}(y)$ is the value in 1994.

Average firm size Data on total employment (`emp`) and total number of firms (`firms`) by firm age bin are from the BDS. Average firm size is the ratio of `emp` to `firms`.

Average earnings In the model, there is no intensive margin of labor supply, so the concept of wages is akin to earnings. To calibrate the wage profile in the model, I target the profile of average earnings-per-employee by firm age group in the QWI data. I use the variable `earns`, which corresponds to average monthly earnings of workers employed for the entire quarter.[5] I construct the average of this series within bins using appropriate employment weights. I average across quarters to obtain a yearly series for each bin.

I then deflate each resulting yearly series by the Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (FRED code: CPIAUCSL). This price index measure uses the years 1982–1984 as the base years. I normalize the units to thousands of dollars so that the units of my resulting average earnings measures are: thousands of 1982–1984 dollars earned per month per worker. I then take an average over the years 1990–1994 within each age bin. The profile across firm age bins is a targeted moment; the profile across worker age bins is a non-targeted moment.

[4] https://www.census.gov/programs-surveys/bds/documentation/methodology.html

[5] See the following link for variable definitions: https://lehd.ces.census.gov/doc/QWI_101.pdf.

Employment share Data on total employment (emp) are from the BDS. Employment share is the ratio of emp within a firm age bin to total emp.

Poaching share The poaching share is number of job-to-job hires as a fraction of total hires. See the following link for variable definitions in the J2J (https://lehd.ces.census.gov/doc/j2j_101.pdf). Job-to-job hires and total hires variables are named j2jhire and mhire, respectively.

Share workers $<$ 45 I use the QWI to construct the age distribution of employment within each firm age bin. I download QWI estimates tabulated by worker sex/age and firm age at the national level (dataset: qwi_us_sa_f_gn_ns_op_u.csv). I select only male workers in the age bins 25–34, 35–44, 45–54, and 55+. I then compute total employment within each firm age bin by summing across worker age bins. The share of workers by age bin within firm age bins is the ratio of total employment by worker age bin, conditional on firm age bin, to total employment within firm age bin. I use the QWI variable emps to get stable measure of employment shares. Data moments are averaged over 1990–1994.

$$\text{Share workers}_{a,f} = \frac{E_{a,f}}{E_f} \quad \text{for worker age bin } a \text{ and firm age bin } f$$

EU separation rate I construct job separation rates in the CPS as described above. The sample includes only male workers age 25 and older. I take averages within age bins over 1990–1994.

Job-to-job flow rate The job-to-job flow rate is simply the quarterly number of direct job-to-job hires (eehire) to average employment over the quarter. Average employment over the quarter is an equally weighted average of beginning of quarter (mainb) and end of quarter (maine) employment.

F.2 Global Optimization Algorithm

Since the parameter space is fairly large and the objective function is not well behaved, I use global methods to find the parameters that minimize the distance between the model and data moments. I use a multiple restart procedure in order to select a set of candidate solutions as starting points and then run a local optimization routine from each of these starting values. The algorithm proceeds as follows:

1. Select a set of $S = 250,000$ candidate starting points using Sobol sequences.

2. Evaluate the objective function at each of these points and store the results in a vector.

3. Keep the best (i.e. lowest function value) $S^* = 1,000$ of these points.

4. Run a local optimization routine (Nelder-Mead algorithm) starting from each of these S^* points and store the resulting function values and parameter vectors.

   (a) Let f^* denote the $1 \times S^*$ vector of objective function values at the local optima corresponding to the S^* starting points.

   (b) Let θ^* denote the $N \times S^*$ matrix of parameter values at the local optima corresponding to the S^* starting points.

5. Find the lowest function value among f^* and call this $\hat{f}$; find the parameter vector in θ^* that corresponds to $\hat{f}$.

6. Let $\hat{\theta}$ denote the parameter vector that corresponds to $\hat{f}$. $\hat{\theta}$ is the global minimum.

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